

Exercise 11

Find the differential of each function.

$$(a) y = xe^{-4x} \qquad (b) y = \sqrt{1-t^4}$$

Solution**Part (a)**

Compute the derivative of y .

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx}(xe^{-4x}) \\ &= \left[\frac{d}{dx}(x) \right] e^{-4x} + x \left[\frac{d}{dx}(e^{-4x}) \right] \\ &= (1)e^{-4x} + x \left[e^{-4x} \cdot \frac{d}{dx}(-4x) \right] \\ &= e^{-4x} + x[e^{-4x} \cdot (-4)] \\ &= e^{-4x}(1 - 4x) \end{aligned}$$

Therefore, the differential of $y = xe^{-4x}$ is

$$dy = e^{-4x}(1 - 4x) dx.$$

Part (b)

Compute the derivative of y .

$$\begin{aligned} \frac{dy}{dt} &= \frac{d}{dt}\sqrt{1-t^4} \\ &= \frac{1}{2}(1-t^4)^{-1/2} \cdot \frac{d}{dt}(1-t^4) \\ &= \frac{1}{2}(1-t^4)^{-1/2} \cdot (-4t^3) \\ &= -\frac{2t^3}{\sqrt{1-t^4}} \end{aligned}$$

Therefore, the differential of $y = \sqrt{1-t^4}$ is

$$dy = -\frac{2t^3}{\sqrt{1-t^4}} dt.$$